JINR-E2-95-27
hep-th/9501100
Jan., 1995

THE FULL STRUCTURE OF QUANTUM $N = 2$ SUPER-$W_3^{(2)}$ ALGEBRA

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Abstract

We present the complete structure of the nonlinear $N = 2$ super extension of Polyakov-Bershadsky, $W_3^{(2)}$, algebra with the generic central charge, c, at the *quantum* level. It contains extra two pairs of fermionic currents with integer spins 1 and 2, besides the currents of $N = 2$ superconformal and $W_3^{(2)}$ algebras. For $c \to \infty$ limit, the algebra reduces to the classical one, which has been studied previously. The 'hybrid' field realization of this algebra is also discussed.

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1. Introduction

Among the *nonlinear* bosonic algebras, there exist very special algebras which contain the bosonic currents with *noncanonical* half-integer spins [1, 2, 3] contrary to other algebras possessing the currents with canonical spins. The Polyakov-Bershadsky, $W_3^{(2)}$, algebra [1, 2] is the simplest nontrivial example of such algebras. It is the bosonic analogue of the *linear* $N = 2$ superconformal algebra (SCA) [4] and includes two bosonic currents with noncanonical spins $3/2$ and two bosonic currents with canonical spins $1, 2$.

Recently [5], its $N = 2$ supersymmetric extension has been constructed at the *classical* level in the sense that one should take into account only single contractions between the composite currents. This algebra comprises, besides the currents of $W_3^{(2)} \propto \{J_w, G^+, G^-, T_w\}$ and $N = 2$ superconformal $\propto \left\{J_s, S, \bar{S}, T_s\right\}$ subalgebras with the same spins $(1, 3/2, 3/2, 2)$ respectively, also additional four fermionic currents $\propto \left\{S_1, \bar{S}_1, S_2, \bar{S}_2\right\}$ with non-canonical integer spins $(1, 1, 2, 2)$: the currents $S, \bar{S}$ are fermionic, their counterparts G^+, G^- are bosonic. There is no intersection of these subalgebras at the embedding in this extended algebra, where all the currents with integer spins can be obtained as the right hand side of operator product expansions (OPEs) between those with half-integer spins.

In this paper, we present the $N = 2$ supersymmetric extension of the $W_3^{(2)}$ algebra at the *quantum* level, by taking into account Jacobi identities to all orders in contractions between the composite currents, and construct explicitly its 'hybrid' field realization on six bosonic and six fermionic fields.

2. The quantum $N = 2$ super-$W_3^{(2)}$ algebra

As subalgebras, $N = 2$ SCA is linear, but $W_3^{(2)}$ algebra is nonlinear. At the quantum level, $N = 2$ SCA is the same as the classical one, but $W_3^{(2)}$ algebra is different from the classical one because of nonlinearity. Bershadsky has found quantum $W_3^{(2)}$ algebra [2] in the context of constrained Wess-Zumino-Witten model. In order to extend the classical consideration [5] to the full quantum version, it is very natural to assume that the *quantum* $W_3^{(2)}$ algebra [2] and $N = 2$ SCA form subalgebras in extended quantum $N = 2$ super $W_3^{(2)}$ algebra.

As we could expect, the algebraic structure of quantum $W_3^{(2)}$ algebra[1] is the same as the classical one except that c dependent coefficients appearing in the right hand side of OPEs are different[2]. (However, this is not the case for $N = 2$ super $W_3^{(2)}$ algebra, as we will see below.) These two subalgebras take the form

$$W_3^{(2)} : \begin{cases} J_w(z_1)J_w(z_2) = \frac{1}{z_{12}^2}\frac{c}{6} \quad , \quad J_w(z_1)T_w(z_2) = \frac{1}{z_{12}^2}J_w, \\ J_w(z_1)G^+(z_2) = -\frac{1}{z_{12}}\frac{1}{2}G^+ \quad , \quad T_w(z_1)G^+(z_2) = \frac{1}{z_{12}^2}\frac{3}{2}G^+ + \frac{1}{z_{12}}G^{+\prime}, \\ T_w(z_1)T_w(z_2) = \frac{1}{z_{12}^4}\frac{(7-6c)c}{(3+2c)} + \frac{1}{z_{12}^2}2T_w + \frac{1}{z_{12}}T_w', \\ G^+(z_1)G^-(z_2) = \frac{1}{z_{12}^3}2c - \frac{1}{z_{12}^2}6J_w + \frac{1}{z_{12}}\left[-\frac{(3+2c)}{(-1+2c)}T_w + \frac{24}{(-1+2c)}J_wJ_w - 3J_w'\right], \end{cases} \tag{1}$$

[1]Normalizations of the currents are different from those of ref. 2.

[2] For example, see the OPE of $G^+(z_1)G^-(z_2)$.

$$
\text{N=2 SCA}: \begin{cases} J_s(z_1)J_s(z_2) = \frac{1}{z_{12}^2}\frac{c}{2} \quad , \quad J_s(z_1)T_s(z_2) = \frac{1}{z_{12}^2}J_s, \\ J_s(z_1)S(z_2) = \frac{1}{z_{12}}\frac{1}{2}S \quad , \\ S(z_1)\bar{S}(z_2) = \frac{1}{z_{12}^3}2c + \frac{1}{z_{12}^2}2J_s + \frac{1}{z_{12}}\left[T_s + J_s'\right], \\ T_s(z_1)S(z_2) = \frac{1}{z_{12}^2}\frac{3}{2}S + \frac{1}{z_{12}}S' \quad , \\ T_s(z_1)T_s(z_2) = \frac{1}{z_{12}^4}3c + \frac{1}{z_{12}^2}2T_s + \frac{1}{z_{12}}T_s'. \end{cases} \tag{2}
$$

where [3] $z_{12} = z_1 - z_2$. Let us also suppose that two $U(1)$ charges of currents for the quantum case (with respect to the currents J_s and J_w) are also the same as in the classical case. Moreover, we assume that all the linear sub-algebras of the classical case do *not* change their structure after passing to the quantum case. Therefore, they have the following form [5]:

$$
\begin{aligned}
& S_1(z_1)\bar{S}_1(z_2) = -\frac{1}{z_{12}^2}\frac{c}{2} + \frac{1}{z_{12}}\frac{1}{2}\left[3J_w - J_s\right], \quad J_s(z_1)S_1(z_2) = -\frac{1}{z_{12}}\frac{1}{2}S_1, \\
& J_w(z_1)S_1(z_2) = -\frac{1}{z_{12}}\frac{1}{6}S_1, \quad J_s(z_1)J_w(z_2) = \frac{1}{z_{12}^2}\frac{c}{3}, \quad J_s(z_1)T_w(z_2) = \frac{1}{z_{12}^2}2J_w, \\
& J_s(z_1)G^+(z_2) = -\frac{1}{z_{12}}G^+, \quad J_s(z_1)S_2(z_2) = -\frac{1}{z_{12}}\frac{1}{2}S_2, \quad J_w(z_1)T_s(z_2) = \frac{1}{z_{12}^2}\frac{2}{3}J_s, \\
& J_w(z_1)S(z_2) = \frac{1}{z_{12}}\frac{1}{3}S, \quad J_w(z_1)S_2(z_2) = -\frac{1}{z_{12}}\frac{1}{6}S_2, \quad G^-(z_1)S_1(z_2) = -\frac{1}{z_{12}}\frac{1}{2}S, \\
& S_1(z_1)\bar{S}(z_2) = \frac{1}{z_{12}}\frac{1}{2}G^+.
\end{aligned} \tag{3}
$$

It is natural to assume that all the remaining OPEs do not change their structure except their structure constants which we should fix from the Jacobi identities. However, we have explicitly checked that the Jacobi identities are not satisfied in this case. In order to get the closed algebra for the quantum case we should add extra terms compared with classical one [4] to the right hand side of OPEs. So we take the most general ansatz consistent with the symmetry under the permutation $z_1 \leftrightarrow z_2$, statistics, spins, and conservations of two $U(1)$ charges. As a result, we arrive at the following OPEs which satisfy the Jacobi identities for the generic value of the central charge

$$
\begin{aligned}
T_s(z_1)T_w(z_2) &= \frac{1}{z_{12}^4}\frac{16c}{3(3+2c)} + \frac{1}{z_{12}^2}\frac{1}{(3+2c)}\left[2T_s - \frac{2(3+2c)}{(-1+2c)}T_w + 8S_1\bar{S}_1 + 8J_sJ_w \right. \\
&\left. + \frac{48}{(-1+2c)}J_wJ_w + 2J_s' - 6J_w'\right] + \frac{1}{z_{12}}\frac{1}{(3+2c)}\left[4S_1\bar{S}_2 + 4S_1\bar{S}_1' - 4S_2\bar{S}_1 - \frac{6}{c}J_sJ_w' \right. \\
&\left. + 4S_1'\bar{S}_1 - \frac{2}{c}J_s'J_s + \frac{2(3+4c)}{c}J_s'J_w + \frac{18}{c}J_w'J_w + 2T_s' + J_s'' - 3J_w''\right], \\
T_s(z_1)G^+(z_2) &= \frac{1}{z_{12}}\frac{2}{(-1+2c)}\left[2S_1\bar{S} - 2J_sG^+ - G^{+\prime}\right], \\
T_s(z_1)S_1(z_2) &= -\frac{1}{z_{12}^2}\frac{1}{2}S_1 + \frac{1}{z_{12}}\frac{1}{2}\left[S_2 - \frac{1}{c}J_sS_1 - \frac{3}{c}J_wS_1 - S_1'\right],
\end{aligned}
$$

[3]Hereafter we do not write down the regular OPEs. All currents appearing in the right-hand sides of the OPEs are evaluated at point z_2. Multiple composite currents are always regularized from the right to the left, unless otherwise stated. Also we have omitted the OPEs which can be obtained through the following automorphism : $J_{w,s} \to -J_{w,s}$, $G^{\pm} \to \pm G^{\mp}$, $S \to \bar{S}$, $\bar{S} \to S$, $S_1 \to \bar{S}_1$, $\bar{S}_1 \to -S_1$,$S_2 \to -\bar{S}_2$, $\bar{S}_2 \to S_2$.

[4] We will explain later that these extra terms disappear in the classical limit.

$$
\begin{aligned}
T_s(z_1)S_2(z_2) &= -\frac{1}{z_{12}^3}\frac{3(1+2c)}{2c}S_1 + \frac{1}{z_{12}^2}\left[S_2 + \frac{3}{2c}J_sS_1 - \frac{9}{2c}J_wS_1 - \frac{3}{2}S_1'\right]\\
&+\frac{1}{z_{12}}\frac{1}{(-1+2c)}\left[2G^+S - \frac{(1+6c)}{2c}J_sS_2 + \frac{(1+6c)}{2c^2}J_sJ_sS_1 + \frac{3(1+2c)}{c^2}J_sJ_wS_1\right.\\
&+\frac{(-1+2c)}{c}J_sS_1' - 4T_sS_1 + \frac{3(-1+2c)}{2c}J_wS_2 - \frac{9(-1+2c)}{2c^2}J_wJ_wS_1\\
&\left.-\frac{3(1+2c)}{c}J_wS_1' + \frac{(-9+2c)}{2c}J_s'S_1 - \frac{3(3+2c)}{2c}J_w'S_1 + \frac{(3+2c)}{2}S_2' - \frac{(-1+2c)}{2}S_1''\right],\\
T_w(z_1)S_1(z_2) &= \frac{1}{z_{12}^2}\frac{(7-6c)}{6(3+2c)}S_1\\
&+\frac{1}{z_{12}}\frac{1}{(3+2c)}\left[\frac{(1-2c)}{2}S_2 + \frac{(-1+2c)}{2c}J_sS_1 - \frac{(3+10c)}{2c}J_wS_1 + \frac{(1-2c)}{2}S_1'\right],\\
T_w(z_1)S(z_2) &= \frac{1}{z_{12}^2}\frac{8}{3(3+2c)}S + \frac{1}{z_{12}}\frac{4}{(3+2c)}\left[-G^-S_1 + J_wS\right],\\
T_w(z_1)S_2(z_2) &= \frac{1}{z_{12}^3}\frac{(-1+2c)(5+6c)}{2c(3+2c)}S_1\\
&+\frac{1}{z_{12}^2}\frac{1}{(3+2c)}\left[\frac{(11+6c)}{3}S_2 - \frac{(5+6c)}{2c}J_sS_1 + \frac{3(5+6c)}{2c}J_wS_1 + \frac{(5+6c)}{2}S_1'\right]\\
&+\frac{1}{z_{12}}\frac{1}{(3+2c)}\left[-2G^+S - \frac{(-1+2c)}{2c}J_sS_2 + \frac{(-1+2c)}{2c^2}J_sJ_sS_1 - \frac{3(1+2c)}{c^2}J_sJ_wS_1\right.\\
&-\frac{(-1+2c)}{c}J_sS_1' - \frac{4(3+2c)}{(-1+2c)}T_wS_1 + \frac{(3+2c)}{2c}J_wS_2 + \frac{3(3+12c+28c^2)}{2c^2(-1+2c)}J_wJ_wS_1\\
&+\frac{3(1+2c)}{c}J_wS_1' - \frac{(-1+2c)}{2c}J_s'S_1 + \frac{(-11+6c)(3+2c)}{2c(-1+2c)}J_w'S_1\\
&\left.+\frac{(-1+2c)}{2}S_2' + \frac{(-1+2c)}{2}S_1''\right],\\
G^+(z_1)S(z_2) &= -\frac{1}{z_{12}^2}2S_1 + \frac{1}{z_{12}}\left[-S_2 + \frac{1}{c}J_sS_1 + \frac{3}{c}J_wS_1 - S_1'\right],\\
G^+(z_1)S_2(z_2) &= -\frac{1}{z_{12}}\frac{(5+6c)}{2(-1+2c)c}G^+S_1,\\
G^-(z_1)S_2(z_2) &= \frac{1}{z_{12}^2}\frac{(5+6c)}{4c}S\\
&+\frac{1}{z_{12}}\left[\frac{(5-2c)}{2c(-1+2c)}G^-S_1 - \frac{1}{2c}J_sS + \frac{3(1+6c)}{2c(-1+2c)}J_wS + \frac{1}{2}S'\right],\\
S_1(z_1)\bar{S}_2(z_2) &= -\frac{1}{z_{12}^3}\frac{1}{2} + \frac{1}{z_{12}^2}\frac{1}{2c}\left[-J_s + 3J_w\right]\\
&+\frac{1}{z_{12}}\left[\frac{1}{2}T_s + \frac{(3+2c)}{2(-1+2c)}T_w + \frac{1}{c}S_1\bar{S}_1 - \frac{1}{2c}J_sJ_s - \frac{3(3+2c)}{2c(-1+2c)}J_wJ_w\right],\\
S(z_1)S_2(z_2) &= \frac{1}{z_{12}}\frac{3}{2c}S_1S,
\end{aligned}
$$

$$
\begin{aligned}
&S(z_1)\bar{S}_2(z_2) = -\frac{1}{z_{12}^2}\frac{3(1+2c)}{4c}G^- \\
&+\frac{1}{z_{12}}\left[\frac{(3+2c)}{2c(-1+2c)}S\bar{S}_1 - \frac{(1+6c)}{2(-1+2c)c}J_sG^- + \frac{3}{2c}J_wG^- - \frac{1}{2}G^{-\prime}\right], \\
&S_2(z_1)S_2(z_2) = \frac{1}{z_{12}}\frac{(1+2c)}{c(-1+2c)}\left[2S_1S_2 - \frac{1}{c}S_1'S_1\right], \\
&S_2(z_1)\bar{S}_2(z_2) = \frac{1}{z_{12}^4}\frac{(-1+4c+6c^2)}{2c} + \frac{1}{z_{12}^3}\frac{(-1+4c+6c^2)}{2c^2}\left[J_s - 3J_w\right] \\
&+\frac{1}{z_{12}^2}\frac{1}{(-1+2c)}\left[\frac{(-1+3c+2c^2)}{c}T_s + \frac{(3+2c)(-1+c-2c^2)}{c(-1+2c)}T_w + \frac{(-1+6c)}{c^2}S_1\bar{S}_1\right. \\
&+\frac{(-2+c)(-1+2c)}{2c^2}J_sJ_s - \frac{3(1+6c)}{c}J_sJ_w + \frac{3(6-5c-4c^2+44c^2)}{2c^2(-1+2c)}J_wJ_w \\
&\left.+\frac{(1+6c)}{2}J_s' - \frac{3(1+6c)}{2}J_w'\right] + \frac{1}{z_{12}}\frac{1}{(-1+2c)}\left[2G^+G^- + \frac{(-1+2c)}{c}S_1\bar{S}_2 + 2S\bar{S}\right. \\
&+\frac{(-1+2c)}{c}S_2\bar{S}_1 + \frac{(-1+2c)}{c}J_sT_s - \frac{(1+2c)(3+2c)}{c(-1+2c)}J_sT_w - \frac{(-1+2c)}{2c^2}J_sJ_sJ_s \\
&-\frac{3(-1+2c)}{2c^2}J_sJ_sJ_w + \frac{3(3+4c+44c^2)}{2c^2(-1+2c)}J_sJ_wJ_w - \frac{3(1+6c)}{2c}J_sJ_w' - \frac{3(1+2c)}{c}J_wT_s \\
&+\frac{3(3+2c)}{c}J_wT_w - \frac{9(3+10c)}{2c^2}J_wJ_wJ_w + \frac{(1+2c)}{c^2}S_1'\bar{S}_1 + \frac{(-1+2c)}{2c}J_s'J_s \\
&-\frac{3(1+6c)}{2c}J_s'J_w + \frac{3(-3+22c)}{2c}J_w'J_w + \frac{(-1+2c)}{2}T_s' - \frac{(3+2c)}{2}T_w' \\
&\left.+\frac{(-1+2c)}{2}J_s'' - \frac{3(-1+2c)}{2}J_w''\right].
\end{aligned}
\tag{4}
$$

The full structure of our algebra can be summarized as (1), (2), (3), (4). Several comments are in order here. We now discuss the relationship between a classical algebra [5] and our quantized version. As we expected, the c-dependent structure constants become more involved rational functions of c. Also one can see that there exist *extra* terms in the right hand side of (4) which do *not* appear in the classical version. The classical limit is given by the usual relation between the Poisson bracket and the commutator while $c \to \infty$ [6]. But we also need to take into account nonlinear terms because of nonlinearity of our algebra. The straightforward way to recover the classical limit is as follows. If we consider any composite current term given by the product of n fields in the right hand side of OPEs at the classical level, then its denominator should be proportional to the $(n-1)$th power of c. So when we effect the $c \to \infty$ limit in any composite current, product of n fields, in quantum algebra, only the term that has the above-mentioned property survives in the classical limit. For example, look at the OPE of $T_s(z_1)T_w(z_2)$ in (4) and consider *only* the terms $[2T_s/(3+2c) - 2T_w/(-1+2c)]/z_{12}^2$ in the right hand side of it. It has "wrong" c-dependence and therefore disappears in the classical limit. Let us remark that all new terms compared to the classical case have the "wrong" c-dependence and after applying these procedures to our OPEs (4), we recover the classical expressions [5].

All the eight currents with spins less than two are primary with respect to the following Virasoro stress-tensor T with zero central charge :

$$T = \frac{1}{1+2c}\left[(-1+2c)T_s + (3+2c)T_w + 8S_1\bar{S}_1 - 8J_s^2 + 24J_wJ_s - 24J_w^2 + 2J_s' - 6J_w'\right], \quad (5)$$

which also reduces to the classical one [5] when $c \to \infty$. T_s and T_w are the quasi primary fields with the central terms equal to $3c$ and $(7-6c)c/(3+2c)$, respectively. However S_2 and $\bar{S}_2$ are not (quasi) primary, they are primary in the following bases :

$$S_2 \to S_2 + \frac{1}{2c}S_1', \quad \bar{S}_2 \to \bar{S}_2 - \frac{1}{2c}\bar{S}_1' \quad (6)$$

It can be checked that in the quantum case also there is no basis in our algebra such that all the currents are primary with respect to any spin two current satisfying the Virasoro algebra.

Notice that the structure constants in the above algebra become divergent if $c = 0, 1/2$, or $-3/2$.

3. 'Hybrid' field realization

Our analysis at the quantum level in this section is basically the same as the one presented in [5]. $N=2$ quantum super-$W_3^{(2)}$ algebra can be realized by the whole multiplets of basic fields containing six bosonic fields - $\left\{U_1, U_2, V_1, V_2, \xi, \bar{\xi}\right\}$ and six fermionic ones - $\left\{\lambda_1, \bar{\lambda}_1, \lambda_2, \bar{\lambda}_2, \psi, \bar{\psi}\right\}$ with the spins $\left(1,1,1,1,\frac{1}{2},\frac{1}{2}\right)$, respectively and with the J_s- and J_w-charges equal to the charges of corresponding currents. The basic fields form the following superalgebra

$$\begin{aligned}
&\xi(z_1)\bar{\xi}(z_2) = -\frac{1}{z_{12}}, \quad \psi(z_1)\bar{\psi}(z_2) = -\frac{1}{z_{12}}, \quad \lambda_1(z_1)\bar{\lambda}_1(z_2) = \frac{1}{z_{12}^2} + \frac{1}{z_{12}}V_1, \\
&\lambda_2(z_1)\bar{\lambda}_2(z_2) = \frac{1}{z_{12}^2} + \frac{1}{z_{12}}V_2, \quad U_2(z_1)V_2(z_2) = -\frac{1}{z_{12}^2}, \quad U_2(z_1)V_1(z_2) = -\frac{1}{z_{12}^2}, \\
&U_1(z_1)V_1(z_2) = \frac{1}{z_{12}^2}, \quad U_1(z_1)\lambda_1(z_2) = \frac{1}{z_{12}}\lambda_1, \quad U_1(z_1)\bar{\lambda}_1(z_2) = -\frac{1}{z_{12}}\bar{\lambda}_1, \\
&U_2(z_1)\lambda_1(z_2) = -\frac{1}{z_{12}}\lambda_1, \quad U_2(z_1)\bar{\lambda}_1(z_2) = \frac{1}{z_{12}}\bar{\lambda}_1, \quad U_2(z_1)\lambda_2(z_2) = -\frac{1}{z_{12}}\lambda_2, \\
&U_2(z_1)\bar{\lambda}_2(z_2) = \frac{1}{z_{12}}\bar{\lambda}_2.
\end{aligned} \quad (7)$$

Taking the most general ansatz for the currents in terms of the defining basic fields and demanding the consistency with the OPEs $(1), (2), (3), (4)$, we can obtain the following realization of our algebra. We only write down the expressions for the basic currents, $S, \bar{S}, G^+, G^-$ because the remaining eight currents can be obtained from the OPEs of basic ones.

$$S = 2^{1/2}\xi\bar{\xi}\psi - \bar{\xi}\bar{\lambda}_2 + \frac{(1-6c)}{2^{3/2}}V_1\psi - \frac{1}{2^{1/2}}V_2\psi + 2^{1/2}U_1\psi + 2^{1/2}U_2\psi + \frac{(-1+2c)}{2^{1/2}}\psi',$$

$$
\begin{aligned}
\bar{S} &= \frac{(1+2c)}{(-1+2c)^2}\xi\lambda_1 + \frac{2^{1/2}}{(1-2c)}\xi\bar{\xi}\bar{\psi} + \frac{1}{2^{1/2}(-1+2c)}V_1\bar{\psi} + \frac{(-3+2c)}{2^{3/2}(1-2c)}V_2\bar{\psi} \\
&+ \frac{2^{1/2}}{(-1+2c)}U_1\bar{\psi} + \frac{1}{2^{1/2}}\bar{\psi}', \\
G^+ &= \bar{\lambda}_1\bar{\psi} + \frac{2^{1/2}}{(1-2c)}\xi\xi\bar{\xi} + \frac{2^{1/2}}{(1-2c)}\xi\psi\bar{\psi} + \frac{2^{1/2}(1+c)}{(-1+2c)}V_1\xi + \frac{(-3+2c)}{2^{3/2}(1-2c)}V_2\xi \\
&+ \frac{2^{1/2}}{(-1+2c)}U_1\xi + \frac{1}{2^{1/2}}\xi', \\
G^- &= (1+2c)\lambda_2\psi + 2^{1/2}\xi\bar{\xi}\bar{\xi} + 2^{1/2}\bar{\xi}\psi\bar{\psi} + \frac{(1-6c)}{2^{3/2}}V_1\bar{\xi} - 2^{1/2}(1+c)V_2\bar{\xi} + 2^{1/2}U_1\bar{\xi} \\
&+ 2^{1/2}U_2\bar{\xi} + \frac{(-1+2c)}{2^{1/2}}\bar{\xi}',
\end{aligned} \tag{8}
$$

The above results (8) are defined up to possible automorphisms of both the $N=2$ quantum $W_3^{(2)}$ algebra and the basic algebras (7). It has been checked that we have correct classical limit [5] in this 'hybrid' field realization through the following automorphism:

$$
\begin{aligned}
&\xi = (2c)^{1/2}\tilde{\xi}, \quad \bar{\xi} = \frac{1}{(2c)^{1/2}}\tilde{\bar{\xi}}, \quad ,\psi = \frac{1}{(2c)^{1/2}}\tilde{\psi}, \quad \bar{\psi} = (2c)^{1/2}\tilde{\bar{\psi}} \\
&\lambda_1 = (2c)^{1/2}\tilde{\lambda}_1, \quad \bar{\lambda}_1 = \frac{1}{(2c)^{1/2}}\tilde{\bar{\lambda}}_1, \quad \lambda_2 = \frac{1}{(2c)^{1/2}}\tilde{\lambda}_2, \bar{\lambda}_2 = (2c)^{1/2}\tilde{\bar{\lambda}}_2
\end{aligned} \tag{9}
$$

It is instructive to examine the structure of the stress-tensor T (5) in this realization. It is bilinear in basic fields,

$$
\begin{aligned}
T = \Big[&-\lambda_1\bar{\lambda}_1 - \lambda_2\bar{\lambda}_2 - \frac{1}{2}\xi\bar{\xi}' + \frac{1}{2}\psi\bar{\psi}' + \frac{1}{2}V_1V_1 + \frac{1}{2}V_2V_2 - V_2U_1 + U_1V_1 - U_2V_2 \\
&+ \frac{1}{2}\xi'\bar{\xi} - \frac{1}{2}\psi'\bar{\psi} + \frac{(1+2c)}{4}V_1' + \frac{(3-2c)}{4}V_2' \Big],
\end{aligned} \tag{10}
$$

and also reduces to its classical version as $c \to \infty$.

4. Conclusion

To summarize, we have constructed the *quantum* $N=2$ super-$W_3^{(2)}$ algebra by using the Jacobi identities for which *extra* composite currents in the right hand side of OPEs, that were *not* present in the classical consideration, are crucial. We have also presented its 'hybrid' field realization. The quantum $N=2$ super-$W_3^{(2)}$ has the same structure as the classical one: it is a closure of quantum $N=2$ SCA and $W_3^{(2)}$ algebra. Thus, the constructed $N=2$ super algebra contains $W_3^{(2)}$ as a genuine subalgebra in contrast to the known $N=2$ superextension of W_3 algebra which yields W_3 only in the limit of vanishing fermionic currents.

Despite the presence of $N=2$ SCA as subalgebra and the equal numbers of bosonic and fermionic currents in $N=2$ quantum super-$W_3^{(2)}$ algebra, the spin contents of currents and OPEs make it impossible to combine *ad hoc* the currents into $N=2$ supermultiplets (the currents of $N=2$ SCA appear in the right hand side of OPEs between other currents). It would be very interesting to study manifestly $N=2$ supersymmetric formulation of this

algebra which allows us to combine the currents into $N=2$ supermultiplets. The main idea of such reformulation is to look for another $N=2$ SCA in the full $N=2$ super-$W_3^{(2)}$ algebra. In the forthcoming papers [7, 8] we will present the corresponding $N=2$ superfield formulations of $N=2$ $W_3^{(2)}$ algebra both at the classical and quantum levels.

Acknowledgments

We would like to thank S.Bellucci, A.Pashnev and especially E.Ivanov for many useful and clarifying discussions and V.Ogievetsky for his interest in this work.

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